Letters

Low Impedance Microstrip Calculations Using MSTRIP

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***Abstract*—The program MSTRIP gives erroneous results for microstrip lines with a characteristic impedance. A modification to the program is suggested.**

TABLE I
MICROSTRIP IMPEDANCE (DIELECTRIC CONSTANT = 2.45 AND NO COVER)

	IMPEDANCE (OHMS)		CAPACITY (PF)	
STRIP WIDTH/ SUBSTRATE HEIGHT	MSTRIP	MODIFIED MSTRIP	MSTRIP	MODIFIED MSTRIP
53 0	4.343	4 338	1184 5	1187 5
63.0	3.669	3.674	1409.3	1405.2
73.0	2.974	3.186	1863.5	1623 9
83.0	2.519	2.811	2295.2	1843.4
93.0	2 196	2.517	2706.8	2060.0
103.0	1 934	2.276	3161.0	2282.6
113 0	1 736	2.080	3584 7	2498.7
123.0	1 576	1 914	4008.7	2715 1
133.0	1.610	1 774	3557 1	2932 1

I. INTRODUCTION

It was found that the program MSTRIP written by Bryant and Weiss [1] gives erroneous results for microstrip lines with a low characteristic impedance. The error is due to the incorrect evaluation of the Green's function.

II. BACKGROUND

In the original program, the Green's function is determined by the integration of the potential function of a source charge on one section of the microstrip. The evaluation of the integral is done in two steps [2]. The first step is a numerical integration of the exact potential function from 0.0 to 6.0 using one hundred points. The second step approximates the Green's function by a cosine integral with limits of 6.0 to infinity. The results of the two calculations are added to give the total Green's function. Unfortunately, there is a cosine term in the potential function that is dependent on the product of the variable of integration, and the distance $B0$ from the source charge to the location where the function is evaluated.

As this distance increases, the number of points required for an accurate evaluation of the first integral increases. For wide strips, one hundred points are not sufficient. The accuracy is improved by making the number of points dependent on the distance ($B0$). However, for very wide strips the required computation time becomes excessive. In addition for good results with wide microstrip it is necessary to increase the total number of charges which requires more evaluations of the Green's function and even more computation time.

III. MODIFIED NUMERICAL INTEGRATION

One possible method of decreasing computation time is to note that for large values of the distance ($B0$), the numerical integration can be done by Gaussian quadrature using cosine weights. The Green's function G can be written as

$$G(B0)=\int_0^{X_1} P(B0,H)\cos(B0\cdot H)\,dH + \int_{X_1}^{\infty} C(B0,H)\,dH \quad (1)$$

where X_1 is chosen to be greater than 6.0, and $P(B0,H)\cdot\cos(B0\cdot H)$ is the potential function of one charge and $C(B0,H)$ is an asymptotic potential function assuming H is large, and H is the Fourier transform variable. It can be shown that [3]

$$\int_0^{\pi} F(\theta)\cos(k\cdot\theta)\,d\theta \approx \frac{1}{k(3\pi/8+\beta)}\sum_{n=0}^{k-1}(-1)^n\left[F\left(\frac{n\pi+\pi/8-\beta}{k}\right) - F\left(\frac{n\pi+7n/8+\beta}{k}\right)\right]$$

with

$$\beta=[3\pi^2-6]^{1/2}-3\pi/8$$

where F is some function of theta and k is a large integer. If k is not an integer, a similar derivation yields

$$\int_0^{(q+1)\pi/k} F(\pi)\cos(k\theta)\,d\theta \approx \frac{1}{k(3\pi/8+\beta)}\sum_{n=0}^{q}(-1)^n\left[F\left(\frac{n\pi+\pi/8-\beta}{k}\right) - F\left(\frac{n\pi+7\pi/8+\beta}{k}\right)\right]. \quad (2)$$

Equation (2) can be used to evaluate the first integral of (1) by replacing k with $B0$ and theta with H. The integer variable q is determined by

$$q \geq \frac{6.0\cdot B0}{\pi}-1$$

and X_1 becomes

$$X_1=\frac{(q+1)\pi}{B0}.$$

It must be remembered that the above substitution is valid only if q is of the order 20 or larger.

IV. MODIFIED PROGRAM

The Green's function subroutine was changed such that the number of integration points was made dependent on $B0$. If the number of points is less than 1000, the original integration

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method is used. If the number of points is greater than 1000, Gaussian quadrature with cosine weights is employed. The number of points is never allowed to be less than 100.

V. Results

Table I compares the output of the original program with the modified one. One possible check for the values in this table is to decompose the microstrip capacity into the parallel plate capacity of the center and the capacity C_f of the ends. The capacity of a wide strip with width $WH1$ would be

$$C=\epsilon_0\epsilon_r(WH1-WH1P)+C_f \qquad (3)$$

where C_f is the capacity of a relatively narrow strip of width $WH1P$ from the original Bryant and Weiss program. Assuming a $WH1P$ of 33.0, (3) becomes

$$C=21.66(WH1-33.0)+751.2 \qquad (\text{pF/m}).$$

The above equation is within 0.5 percent with of values in Table I.

References

[1] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.
[2] ——, "Dielectric Green's function for parameters of microstrip," *Electron Lett.*, vol. 6, pp. 462–464, 560, 1974.
[3] F. B. Hildebrand, *Introduction to Numerical Analysis*. New York: McGraw-Hill, 1974, 427–432.

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